

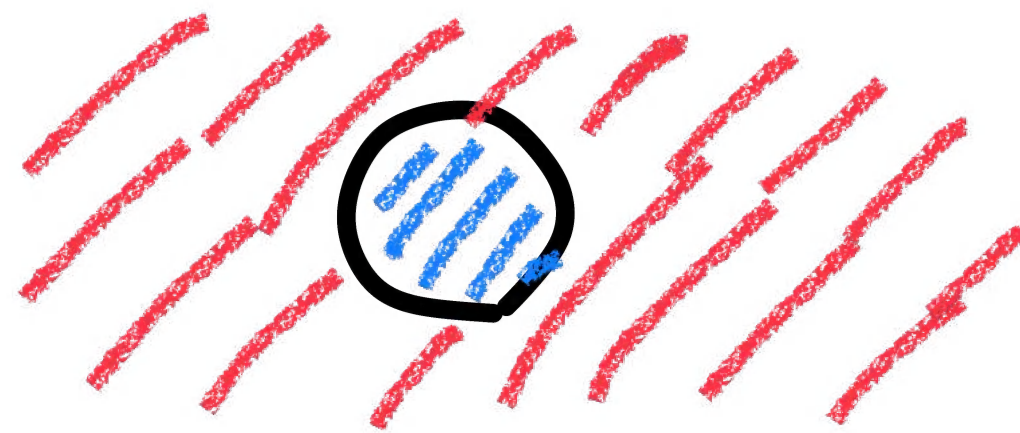
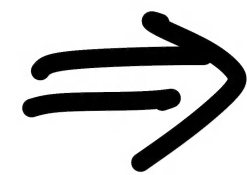
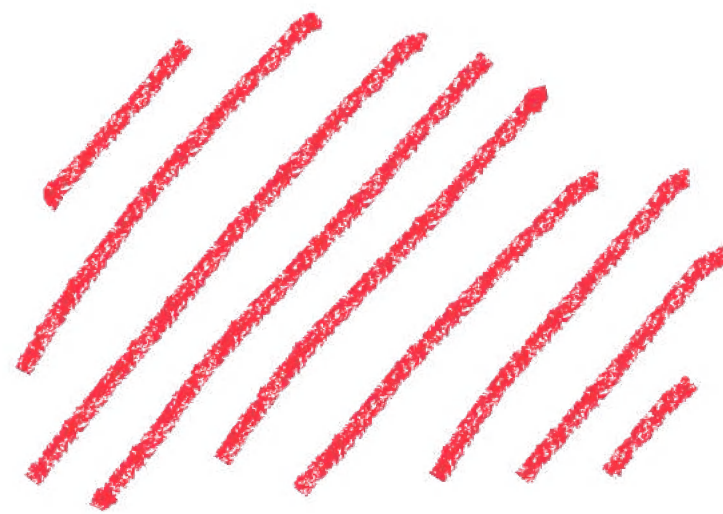
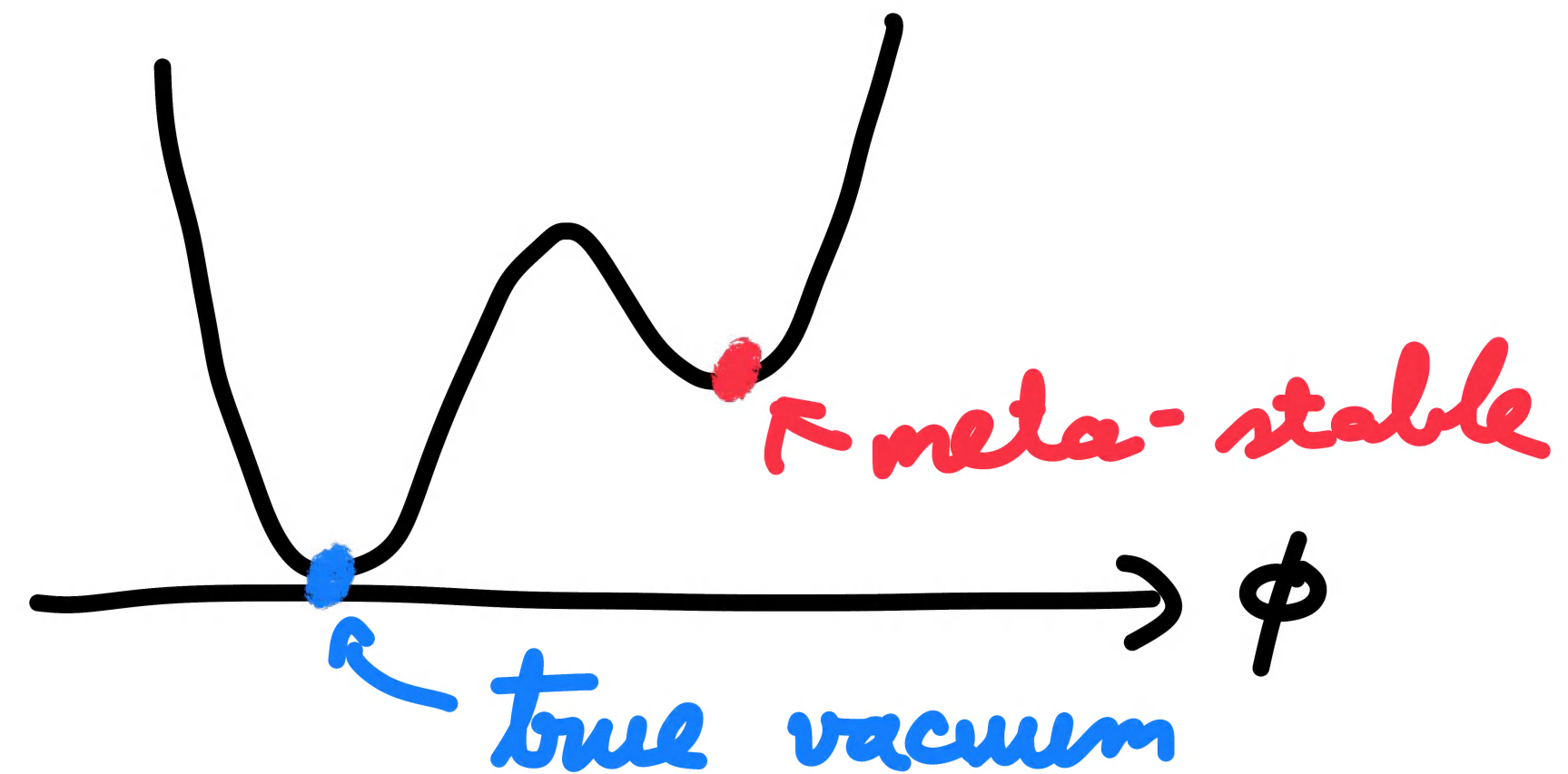
Modified instanton sum in 4d gauge theories

Yuya Tanizaki (YITP, Kyoto)

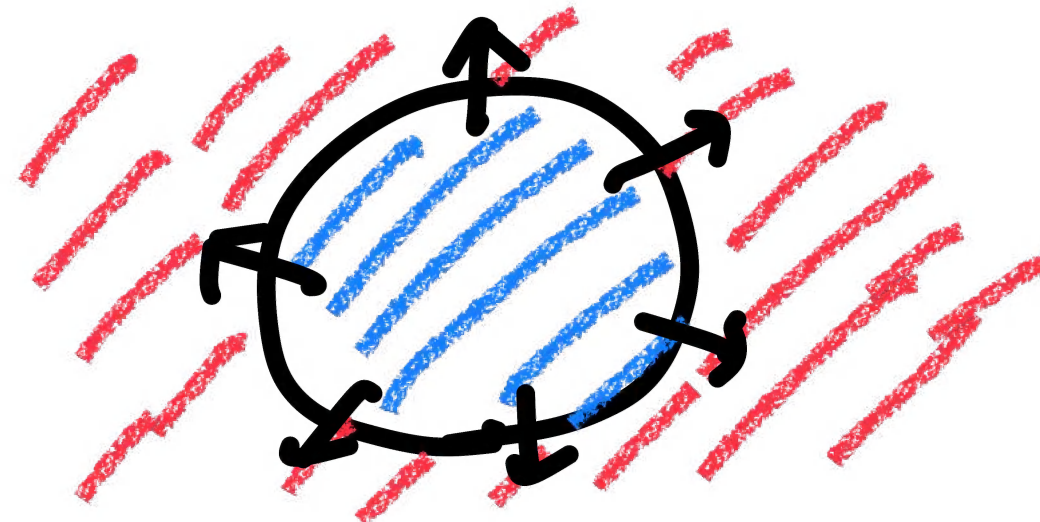
1912.01033 w/ Mithat Ünsal (NCSU)

Decomposition 2021

Vacuum decay



bubble of
true vacuum



expansion
of bubble

This may sound to be general.

But, this *never occurs* if QFT enjoys "decomposition".

Example : 2d Maxwell theory

$$\mathcal{L} = \frac{1}{2e^2} F_{01}^2 + \frac{\theta}{2\pi} F_{01} \quad (\text{in real time})$$

Quantization (in temporal gauge)

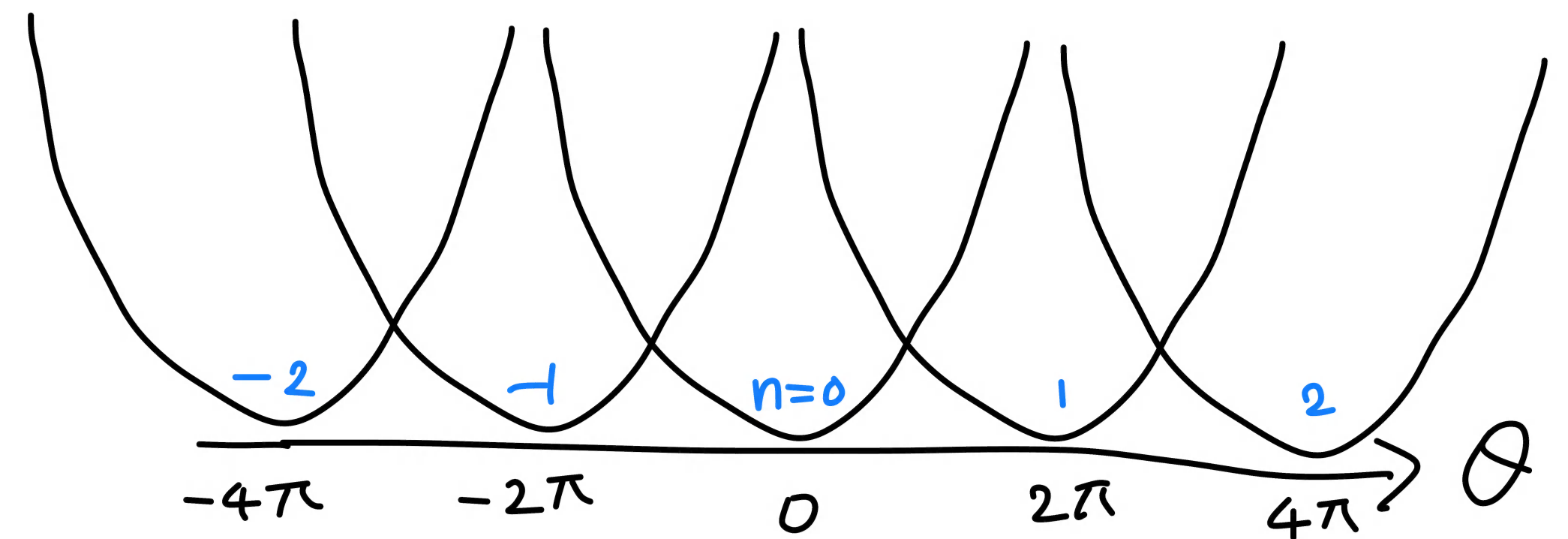
$$\left(\pi = \frac{\partial \mathcal{L}}{\partial \dot{a}_1} = \frac{1}{e^2} \dot{a}_1 + \frac{\theta}{2\pi} \right) \quad (\text{canonical momentum})$$

$$\left(H = \frac{e^2}{2} \left(\pi - \frac{\theta}{2\pi} \right)^2 \right) \quad (\text{Hamiltonian})$$

w/ Gauss law $\partial_1 \pi = 0$.

Eigenstates (on $S^1 \times \mathbb{R}_{\text{time}}$)

$$\begin{cases} \Psi_n[A_1] = \exp\left(in \oint_0^L dx a_1(x)\right) \\ E_n = \frac{e^2}{2} \left(n - \frac{\theta}{2\pi}\right)^2 \end{cases}$$



Ψ_n is an eigenstate. So, it never decays to the lowest energy state !!

Just an accident for a free theory ??

Or, any other reasonings ??

↳ 1-form symmetry in (1+1) dim.

Symmetry generator of $U(1)^{[1]}$

$$U_\alpha(x) = \exp(i\alpha \Pi(x)).$$

This is a topological point-like operator.

$$\left(\begin{array}{l} \partial_1 U_\alpha(x) = (\partial_1 \Pi) U_\alpha(x) = 0 \quad (\text{Gauss law}) \\ \partial_t U_\alpha(x) \sim [H, U_\alpha(x)] = 0 \quad (H \sim \Pi^2) \end{array} \right) \text{---} (*)$$

Each state $|n\rangle = \mathbb{P}_n$ is an eigenstate of this operator:

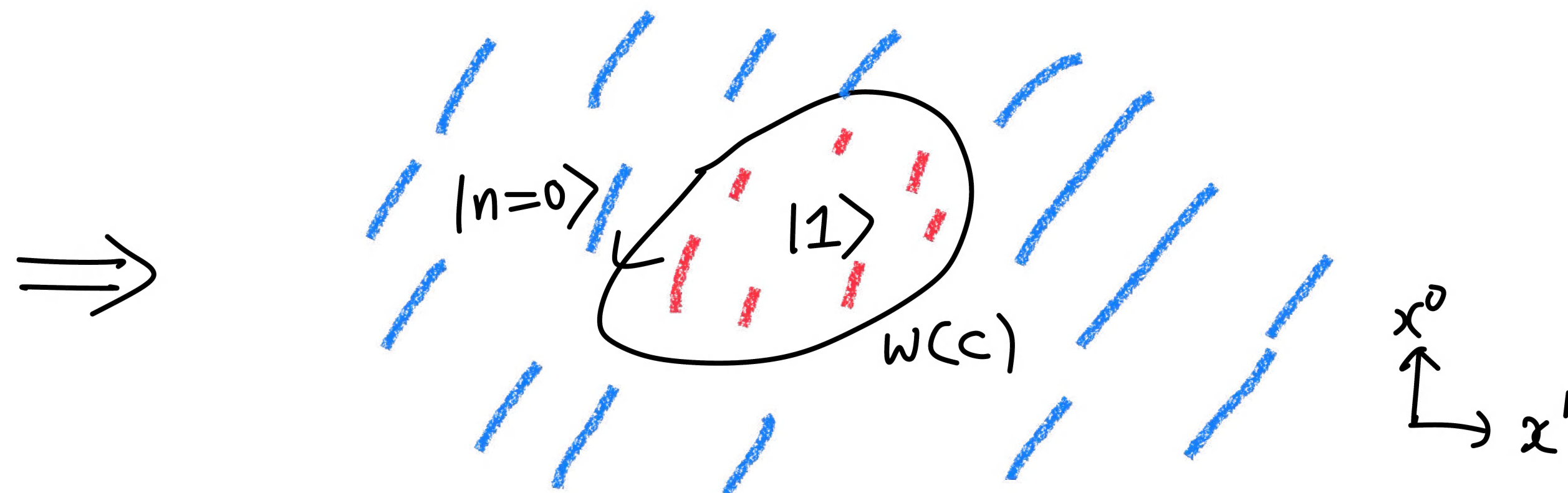
$$U_\alpha(x) |n\rangle = e^{i\alpha n} |n\rangle.$$

Conservation law (*) tells that $U_\alpha(x)$ should be constant in spacetime !!

Wilson loop connects $|n\rangle$ and $|n\pm 1\rangle$.

$$W(C) = \exp(i \oint_C a)$$

$$\oint_C U_a = \oint_C 1 \cdot e^{i\alpha}$$



But, local operators in QFT have no ways to change n .

So, no decays.

Generalization to interacting theories

2d $U(1)$ gauge theory w./ charge- p matter ($p > 1$):

$$\mathcal{L} = \frac{1}{2e^2} F_{01}^2 + \frac{\theta}{2\pi} F_{01} + \bar{\Psi} i \gamma^\mu (\partial_\mu + i p A_\mu) \Psi + m \bar{\Psi} \Psi.$$

$U(1)^{[1]}$ is explicitly broken. But, it still enjoys $\mathbb{Z}_p^{[1]} \subset U(1)^{[1]}$, generated by

$$U(x) \sim \exp\left(\frac{2\pi i}{p} \Pi(x)\right).$$

\Rightarrow Some vacuum decay is possible, such as $|n \pm p\rangle \Rightarrow |n\rangle$.

$$|p\rangle = \frac{U = (e^{\frac{2\pi i}{p}})^p = 1}{|p\rangle} \rightarrow x' \Rightarrow \begin{array}{c} U=1 \\ \leftarrow -p \\ (p) \end{array} \xrightarrow{U=1} \begin{array}{c} +p \\ (0) \end{array} \xrightarrow{U=1} |0\rangle \Rightarrow |0\rangle \quad \checkmark$$

Still, $|n=1\rangle \Rightarrow |n=0\rangle$ is NOT allowed:

$$\frac{U=1}{|0\rangle} \xrightarrow{U=e^{2\pi i/p}} |1\rangle \rightarrow x \quad \times$$

Generalization to higher dim

So far, everything is about 2dim QFTs.

Higher dimensions ??

\Rightarrow $(d-1)$ -form symmetry. (YT, Ünsal, 2019)

There should be a topological and point-like operator $U(x)$.

Universe

"Decompose" the Hilbert space by eigenvalues of $U(x) : (\mathbb{Z}_p^{[d-1]})$

$$\mathcal{H} = \bigoplus_{k=0}^{p-1} \mathcal{H}_k \quad \left(\mathcal{H}_k = \{ |\psi\rangle \in \mathcal{H} \mid U(x)|\psi\rangle = e^{i\frac{2\pi}{p}k} |\psi\rangle \} \right)$$

Let's call each \mathcal{H}_k as universe.

Vacuum decay / domain walls between different \mathcal{H}_k are not possible.

4 dim Yang-Mills theory

$$S = \frac{1}{g^2} \int_{M_4} \text{tr}(F \wedge *F) + i \int_{M_4} \frac{\theta}{8\pi^2} \text{tr}(F \wedge F).$$

Let's quantize this theory on compact spacetime M_4 :

$$Z = \int \mathcal{D}a \exp(-S).$$

As $\pi_3(SU(N)) = \mathbb{Z}$, while $\pi_1 = \pi_2 = 0$, there are nontrivial bundles w/

$$n = \frac{1}{8\pi^2} \int_{M_4} \text{tr}(F \wedge F) \in \mathbb{Z}.$$

Q. Do we have to sum up all these distinct sectors?

A. since '70s Yes. This is required for locality / cluster decomposition.

A. No. Modified instanton sum is also consistent.

Modified instanton sum in 4d YM

Let us declare that we only sum up restricted instanton sectors

$$\frac{1}{8\pi^2} \int_{M_4} \text{tr}(F \wedge F) \in p \mathbb{Z}.$$

This is consistent with locality.

Indeed, there is a local Lagrangian!! (Seiberg, 2010)

$$\mathcal{L} = \frac{1}{g^2} \text{tr}(F \wedge *F) + i \frac{\theta}{8\pi^2} \text{tr}(F \wedge F) \\ + 2\pi i \frac{\chi^{(0)}}{2\pi} \wedge \left(\frac{1}{8\pi^2} \text{tr}(F \wedge F) - \frac{p}{2\pi} dC^{(3)} \right)$$

$$\begin{cases} \chi^{(0)} : 2\pi\text{-periodic scalar.} \\ C^{(3)} : U(1) \text{ 3-form gauge field.} \end{cases}$$

$$\text{EOM of } \chi^{(0)} : \frac{1}{8\pi^2} \text{tr}(F \wedge F) = \frac{p}{2\pi} dC^{(3)} \quad \Rightarrow \quad \int \frac{1}{8\pi^2} \text{tr}(F^2) \in p \mathbb{Z}.$$

3-form symmetry & universe

Instead of EOM of $\chi^{(0)}$, consider EOM of $c^{(3)}$:

$$d\chi^{(0)} = 0.$$

$\Rightarrow U(x) = \exp(i\chi^{(0)}(x))$ is a topological, point-like operator,
so is a generator of 3-form symmetry.

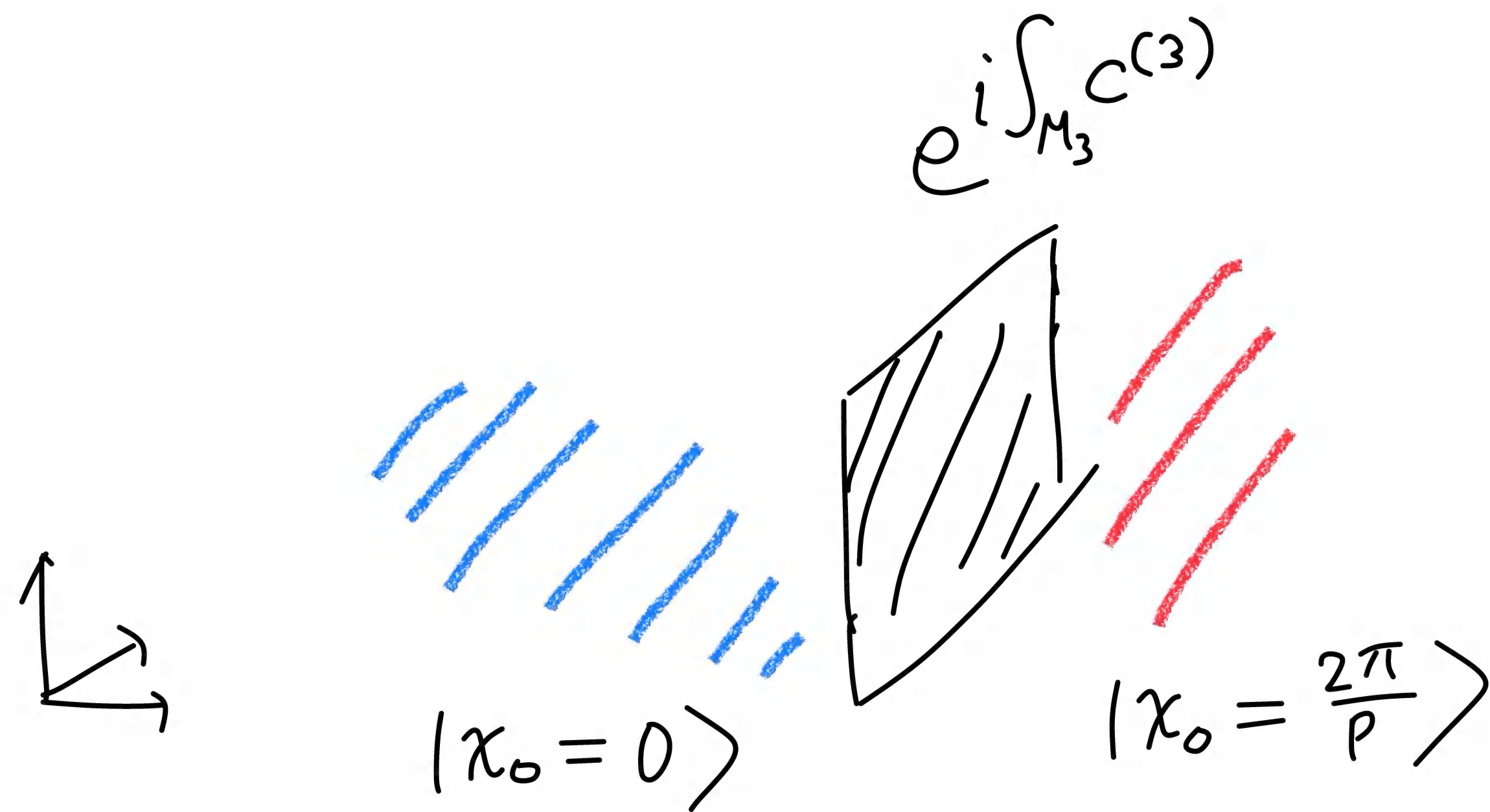
Moreover,

$$\int_{\partial c^{(3)}} e^{i p \int \chi^{(0)} \wedge d c^{(3)}} = \sum_n e^{i p n \chi^{(0)}} = \delta\left(\chi^{(0)} \in \frac{2\pi}{p} \mathbb{Z}\right).$$

$\Rightarrow U(x)^p = 1$. Thus, $U(x)$ generates $\mathbb{Z}_p^{[3]}$.

$$\mathcal{H} = \bigoplus_{k=0}^{p-1} \mathcal{H}_k \quad \text{w/} \quad \mathcal{H}_k = \left\{ |\psi\rangle \mid e^{i\chi^{(0)}(x)} |\psi\rangle = e^{\frac{2\pi i}{p} k} |\psi\rangle \right\}.$$

Different universes can be connected only if Wilson "wall" operator is inserted:



When $|\chi_0 = 0\rangle$, $|\chi_0 = \frac{2\pi}{p}\rangle$ have different energies,
this is a would-be domain wall for vacuum decays.

But, it is never dynamically generated due to 3-form sym.

1-form + 3-form symmetry \Rightarrow 4-group symmetry

$SU(N)$ YM theory also enjoys $\mathbb{Z}_N^{[1]}$, acting on Wilson loops.

However, gauging of $\mathbb{Z}_N^{[1]}$ is not possible:

$B: \mathbb{Z}_N$ 2-form gauge field $F \Rightarrow F - B$.

$$\text{Then, } Q_{\text{top}} = \frac{1}{8\pi^2} \int \text{tr}((F - B)^2) = - \underbrace{\frac{N}{8\pi^2} \int B \wedge B}_{\in \frac{1}{N} \mathbb{Z}} \pmod{1}.$$

However, EOM of $\chi^{(0)}$ says $Q_{\text{top}} = \frac{p}{2\pi} \int dc^{(3)} \in p \mathbb{Z}$.

To make it consistent, we must also introduce

$D: \mathbb{Z}_{pN}$ 4-form gauge field.

w/

$$\underline{\frac{p}{2\pi} D = - \frac{N}{8\pi^2} B \wedge B \pmod{1}}$$

4 Hooft anomaly / global inconsistency

In modified YM, as $\frac{1}{8\pi^2} \text{tr}(F^2) \in p\mathbb{Z}$,

$$\theta \sim \theta + \frac{2\pi}{p}.$$

Introducing the background 2-form & 4-form gauge field (B, D) ,

$$Z_{\theta + \frac{2\pi}{p}}[B, D] = e^{i \int D} \times Z_{\theta}[B, D].$$

"anomaly" bet. different universes

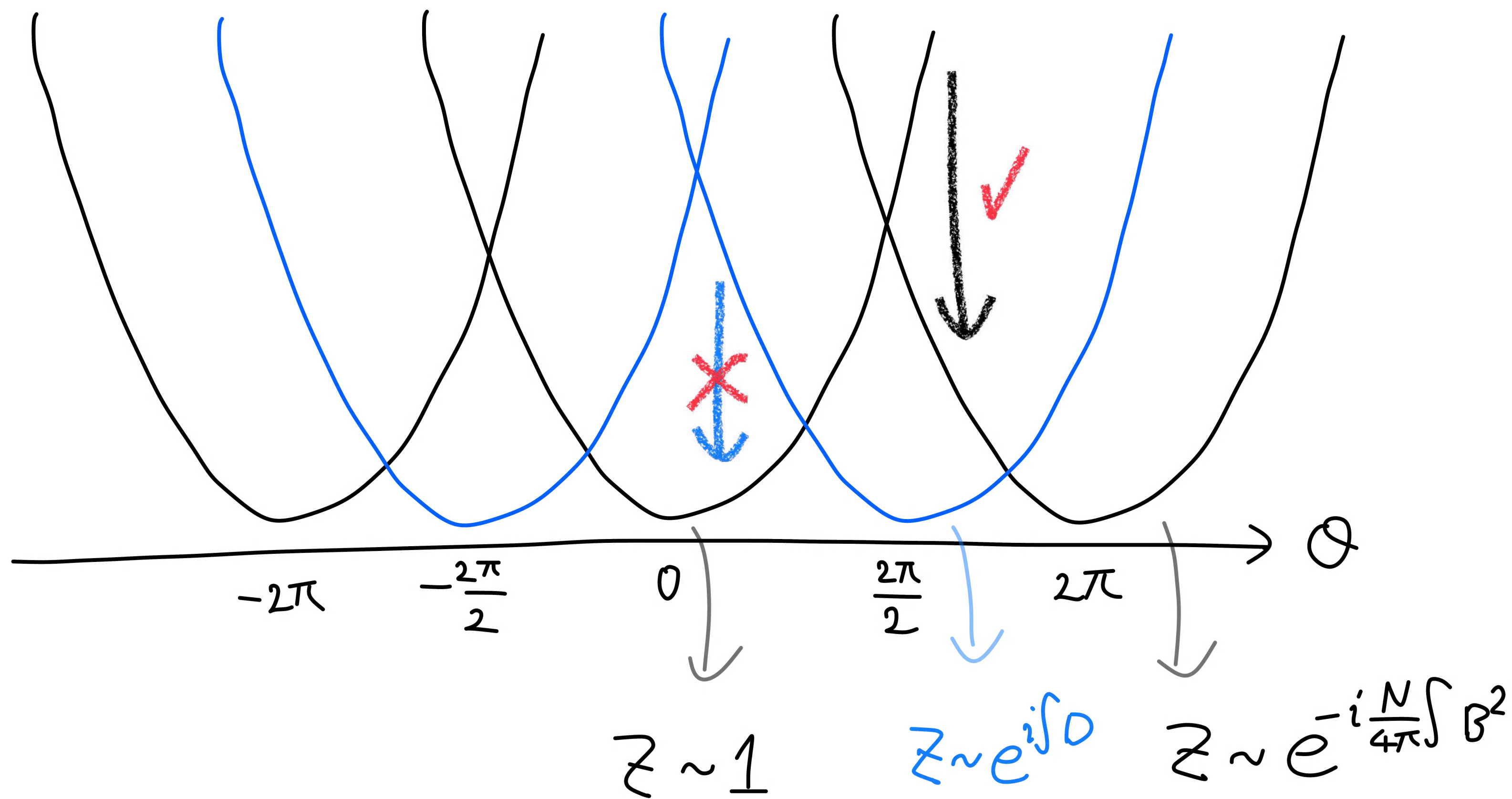
Thanks to the constraint, D can be eliminated for $\theta \rightarrow \theta + 2\pi$:

$$\begin{aligned} Z_{\theta + 2\pi}[B, D] &= e^{i p \int D} \times Z_{\theta}[B, D] \\ &= e^{-i \frac{N}{4\pi} \int B \wedge B} \times Z_{\theta}[B, D]. \end{aligned}$$

"anomaly" within the same universe

$$\underline{p=2}$$

$$\mathcal{H} = \mathcal{H}_0 \oplus \mathcal{H}_1$$



Fermions & chiral symmetry w/ modified instanton sum

Let's add a massless Dirac fermion.

Usually, its chiral symmetry is absent due to ABJ anomaly:

$$\bar{\psi} \psi \rightarrow \exp\left(2i\alpha \frac{1}{8\pi^2} \int \text{tr}(F \wedge F)\right) \bar{\psi} \psi$$

for $\psi \rightarrow e^{i\alpha \gamma_5} \psi$, $\bar{\psi} \rightarrow \bar{\psi} e^{i\alpha \gamma_5}$.

\Rightarrow This is symmetry only if $\alpha \in \pi \mathbb{Z}$ for the usual instanton sum,
so only the fermion parity (vector symmetry!) survives.

In modified instanton sum, $\frac{1}{8\pi^2} \int \text{tr}(F^2) \in p \mathbb{Z}$, $\alpha \in \frac{2\pi}{2p} \mathbb{Z}$ gives symmetry.

$\Rightarrow (\mathbb{Z}_{2p})_{\text{axial}}$ is a good symmetry.

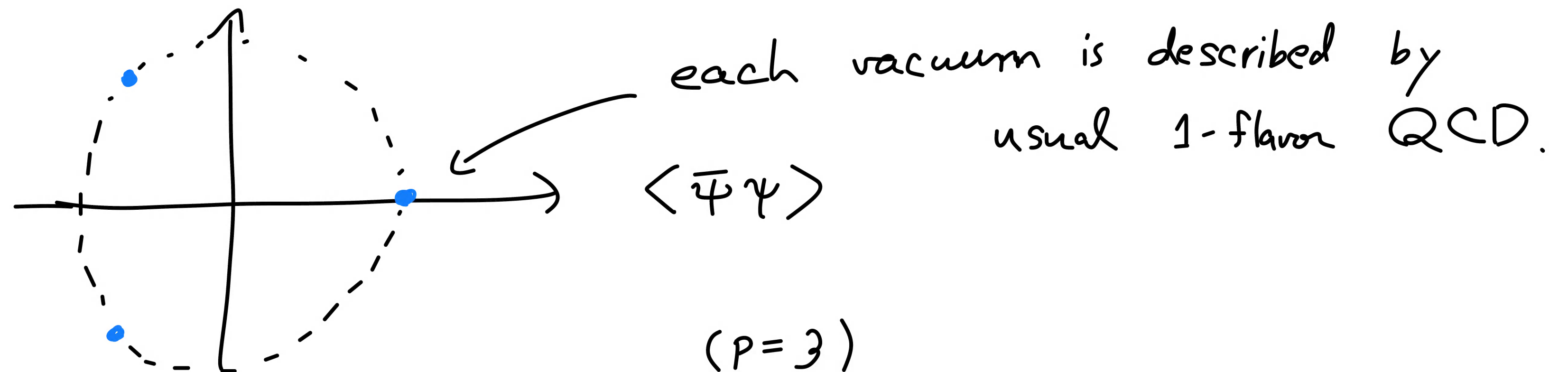
Spontaneous breaking $(\mathbb{Z}_{2p})_{\text{axial}} \rightarrow (\mathbb{Z}_2)_F$

$(\mathbb{Z}_{2p})_{\text{axial}}$ & $\mathbb{Z}_p^{[3]}$ has a mixed anomaly:

$$S_{5\text{dim}} = \frac{i}{2\pi} \int_{M_5} (2A_{\text{axial}}) \wedge D.$$

\Rightarrow To match the 't Hooft anomaly, we need SSB

$$(\mathbb{Z}_{2p})_{\text{axial}} \longrightarrow (\mathbb{Z}_2)_F.$$



Summary

- Decomposition of QFT is a general phenomenon, when the d -dim. QFT has a $(d-1)$ -form symmetry.
 - $\mathcal{U}(x)$: a topological, point-like op.
 - Universe is distinguished by eigenvalues of $\mathcal{U}(x)$.
- In 4d YM, $\mathbb{Z}_p^{[3]}$ appears by restricting instanton sectors into $Q_{\text{top}} \in p\mathbb{Z}$.
- For $SU(N)$ pure YM, $\mathbb{Z}_N^{[1]}$ and $\mathbb{Z}_p^{[3]}$ form a 4-group structure:
$$p D_{4\text{-form}} = -\frac{N}{4\pi} B_{2\text{-form}}^2 \pmod{1}.$$
- It has a mixed anomaly w/ Θ -periodicity and/or chiral symmetry.